event (these being the well-determined quantities). The configurations differ from one another in beam momentum which we assume determined only up to a probability distribution. The weighted sum is then taken over this parameter. We assume the probability distribution to be of Gaussian form, centered at the nominal value, with a width consistent with our assumption of a beam momentum of 440 ± 25 MeV/c at the copper plate. The weighted sum is taken from -2σ to $+2\sigma$, where σ is the standard deviation of the Gaussian. We find that this correction reduces our odds on $\xi=0$ versus $\xi=-6.5$ by less than an order of magnitude.

References to existing measurements of ξ for the decays $K^+\rightarrow\mu^++\pi^0+\nu$ and $K_2{}^0\rightarrow\mu^\pm+\pi^\mp+\nu(\bar{\nu})$ are given in Refs. 5–11. Although the early results[5–7] seem to be in disagreement, the more recent values[8–11] all appear to be consistent with the value $\xi=0$. In conclusion we find that to the extent that we have been able to test the decay $K^-\rightarrow\mu^-+\pi^0+\bar{\nu}$, the form factor behavior is quite consistent with what has been reported recently on the decays $K^+\rightarrow\mu^++\pi^0+\nu$ and $K_2{}^0\rightarrow\mu^\pm+\pi^\mp+\nu(\bar{\nu})$.

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Solutions in Pion-Pion Scattering for Two ρ Mesons*

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The theory of pion-pion scattering with inelastic effects is applied to the experimental evidence for the B particle which is interpreted as a second resonance in the $J=1^-$ pion-pion scattering amplitude. The full width of the B particle is predicted and the positions of the ρ and B particles are found to be consistent with the assumption that the $\pi-\omega$ channel dominates the inelastic intermediate state.

SOME time ago, we discussed solutions of the low-energy pion-pion scattering problem with inelastic contributions to the intermediate states.[1,2] The most interesting feature of these solutions was the appearance of two resonances with the same quantum numbers when the inelastic pion-pion scattering was small. It is interesting to reexamine this result in the light of the recent experimental discovery of the B particle, which suggests a companion resonance to the ρ meson. This new resonance ρ' is conjectured to have the decay channels $\rho'\rightarrow\pi+\pi(f^0)$ and $\rho'\rightarrow\pi+\omega(B)$ and its quantum numbers are $J=1$, $P=-1$, and $G=+1$ as for the ρ meson.[4] In this note, we wish to relate this situation to our previous results and to find the conditions under which two resonances with these masses could be expected. Our main result is the prediction of the ratio of the total widths of the ρ and ρ'.

The basic conclusions of the two previous papers[1,2] are as follows: If there exists a solution with inelastic intermediate states set equal to zero and which exhibits a resonance in the P wave, then if small inelastic contributions are introduced with a threshold just below this resonance, a second resonance will appear at a

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higher mass value and the initial resonance is depressed to a lower mass. Above the inelastic threshold ν_T the phase shift $\delta=\delta_R+i\delta_I$, and let us suppose that δ_R passes through π at $\nu=\nu_\pi$ and that δ_I is small. Then from scattering theory $R=1+\sigma^{\text{in}}/\sigma^{\text{el}}$ can be written

$$R\approx[\delta_I/\{(\delta_R-\pi)^2+\delta_I^2\}]+1\,,\quad(\delta_R-\pi\ll\delta_I)\tag{1}$$

and the P-wave amplitude f as a function of ν (=c.m. momentum squared) is given by

$$f^{-1}=\frac{A}{\nu}(\nu_\pi-\nu)-i\left(\frac{\nu}{\nu+1}\right)^{1/2}-\frac{1}{\beta(\nu_\pi-\nu)-i\kappa(\nu-\nu_T)^{1/2}}\,,\tag{2}$$

where

$$\delta_I\approx\alpha=\kappa(\nu-\nu_T)^{1/2}\,,$$
$$\delta_R-\pi\approx\beta(\nu-\nu_\pi)\,.\tag{3}$$

The last term in (2) is the inelastic contribution and this term produces a Castillejo, Dalitz, Dyson[5] (CDD) pole in the unphysical sheet in the $\nu-\nu_T$ plane at $\nu_P\approx\nu_\pi-i\alpha/\beta$ assuming $\alpha\ll\beta$. A is a parameter which is insensitive to the inelastic cut in the physical region. In previous low-energy pion-pion calculations[6] it was found for the coupling constant $\lambda=-0.1$ that $\nu_\pi=4.6$ and $A=29$. As was shown in Ref. 2, the two resonances will be at

$$\nu_{R_{1,2}}\approx\nu_\pi\mp(\nu_\pi/A\beta)^{1/2}\,,\tag{4}$$

if we assume that κ is small.

Let us now examine the present experimental situation. The two resonances are at 750 and 1250 MeV, so that from Eq. (16) of Ref. 2 we get

$$\nu_{R_1}=6.2\quad\text{and}\quad\nu_{R_2}=18.9\,,\tag{5}$$

and therefore $\nu_\pi=12.5$. This value of ν_π corresponds to a somewhat smaller value of $|\lambda|$ than previously considered. We can calculate A from the width of one of the resonances. Using Eqs. (14) and (16) of Ref. 2, we find that the width Γ is given by

$$\Gamma=\mu\{[\nu_R/(\nu_R+1)]^{1/2}\nu_R/2A\}^{1/2}\,,\tag{6}$$

where ν_R is the position of the resonance and μ is the pion mass. Given Γ for one resonance, we may find A and hence Γ for the other resonance. Assuming $\Gamma_\rho=100$ MeV we find that $A=5.6$ and $\Gamma_{\rho'}=180$ MeV.

We found in our previous work[1,2] that the lower resonance at ν_{R_1} usually occurred just below the first inelastic threshold of any consequence. Thus, in the case of the ρ and ρ' mesons the inelastic threshold ν_T should be between $\nu=8$ and $\nu=12$, and probably corresponds to the onset of the $\pi+\omega$ intermediate state at $\nu=11$.

In summary, provided the inelastic cross section does not become too large for energies up to 1.5 BeV, then the ρ and ρ' mesons can be explained in pion-pion scattering by solutions of the analyticity, unitarity and crossing equations which, in the presence of inelastic scattering, develop a CDD pole on the unphysical sheet.

Note added in proof. It now appears that the f^0 cannot be $J=1^-$ since the decay $f^0\rightarrow 2\pi^0$ has been observed. However, the argument of this paper still holds for the B particle.

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